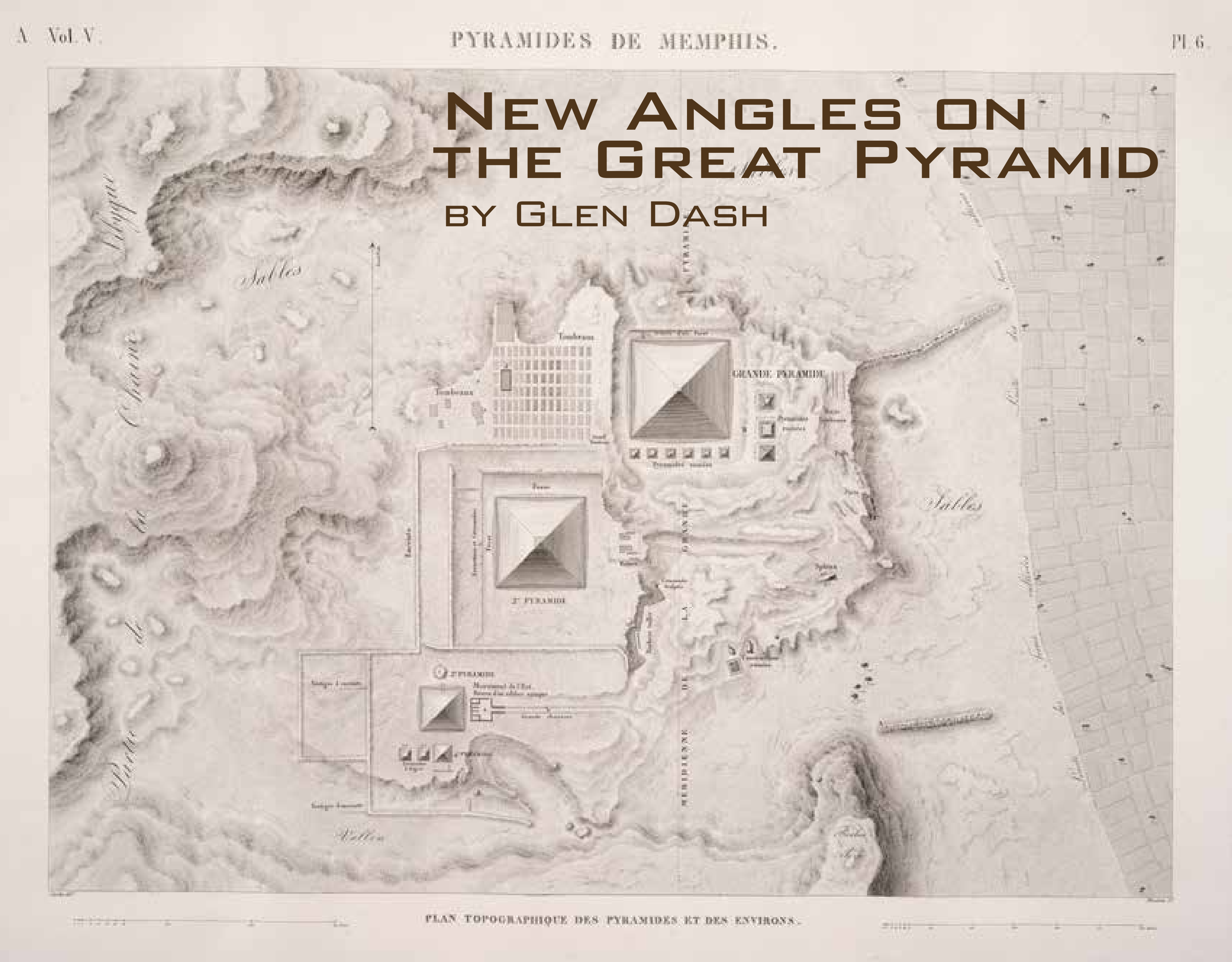


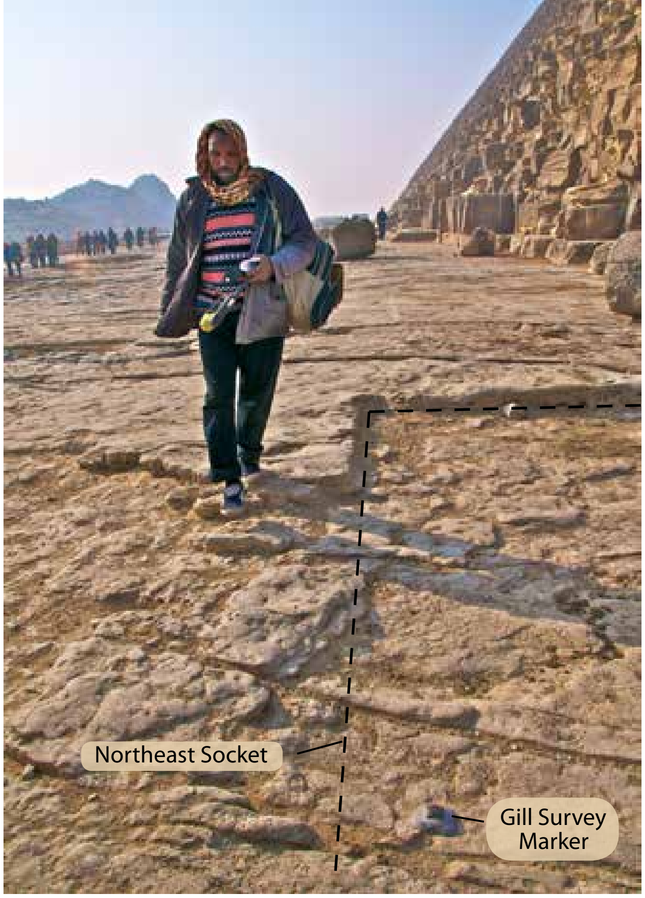
New Angles on the Great Pyramid

by Glen Dash

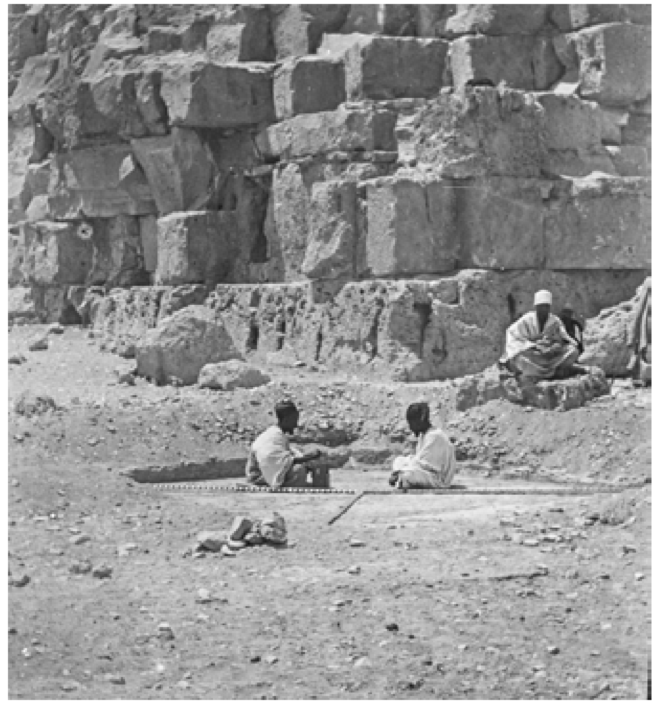
In 1984 Mark Lehner and David Goodman measured the elusive base of the Great Pyramid of Khufu. They followed in the footsteps of researchers, going back to the 17th century, who tried to determine the true dimensions of the pyramid—no easy task. Stripped of nearly all of its casing, the monument no longer has any corners, nor well-defined edges. Now, for the first time, we publish the Lehner-Goodman data with an analysis that gives the dimensions and orientation of the Great Pyramid.

No monument in the world has given rise to more speculation about its meaning than the Great Pyramid of Khufu. It has been said to encode "God's unit of measurement"— the Pyramid inch—to physically represent the mathematical constant pi, and incorporate the Golden Section. Sir Isaac Newton thought it could be used to refine his theory of universal gravitation. All of these ideas, sensible or not, depended to one degree or another on knowing the exact size and orientation of the Great Pyramid. It is surprising then to find that there has been no final, definitive work on the subject. The reason is due, in large part, to the condition we find the Pyramid in today. We find scant traces of its original corners. The best we can do is to project their original positions from the fragmentary data that does remain. It has proven to

Above: Map of the Giza Plateau prepared by Napoleon's expedition. *Description de l'Égypte, Antiquites Planches*, *vol 5. Pyramids de Memphis.* Plate 6, Plan Topographique des Pyramides et des Environs, 1809–17. Image courtesy of the Linda Hall Library of Science, Engineering & Technology. Right: David Goodman surveys in 1984 with a theodolite and electronic distance measuring device to establish the Giza Plateau Mapping Project grid, looking toward the Great Pyramid. View to the east-northeast. Photo by Mark Lehner.



Mark Lehner



The Northeast Corner Socket. Left: AERA Surveyor Mohammed el-Baset walks past the remains of the socket (dotted line). Right: The same corner socket was photographed by Piazzi Smyth in 1865. Smyth photo © Photoarchive3D; courtesy of George Mutter and Bernard Fishman.

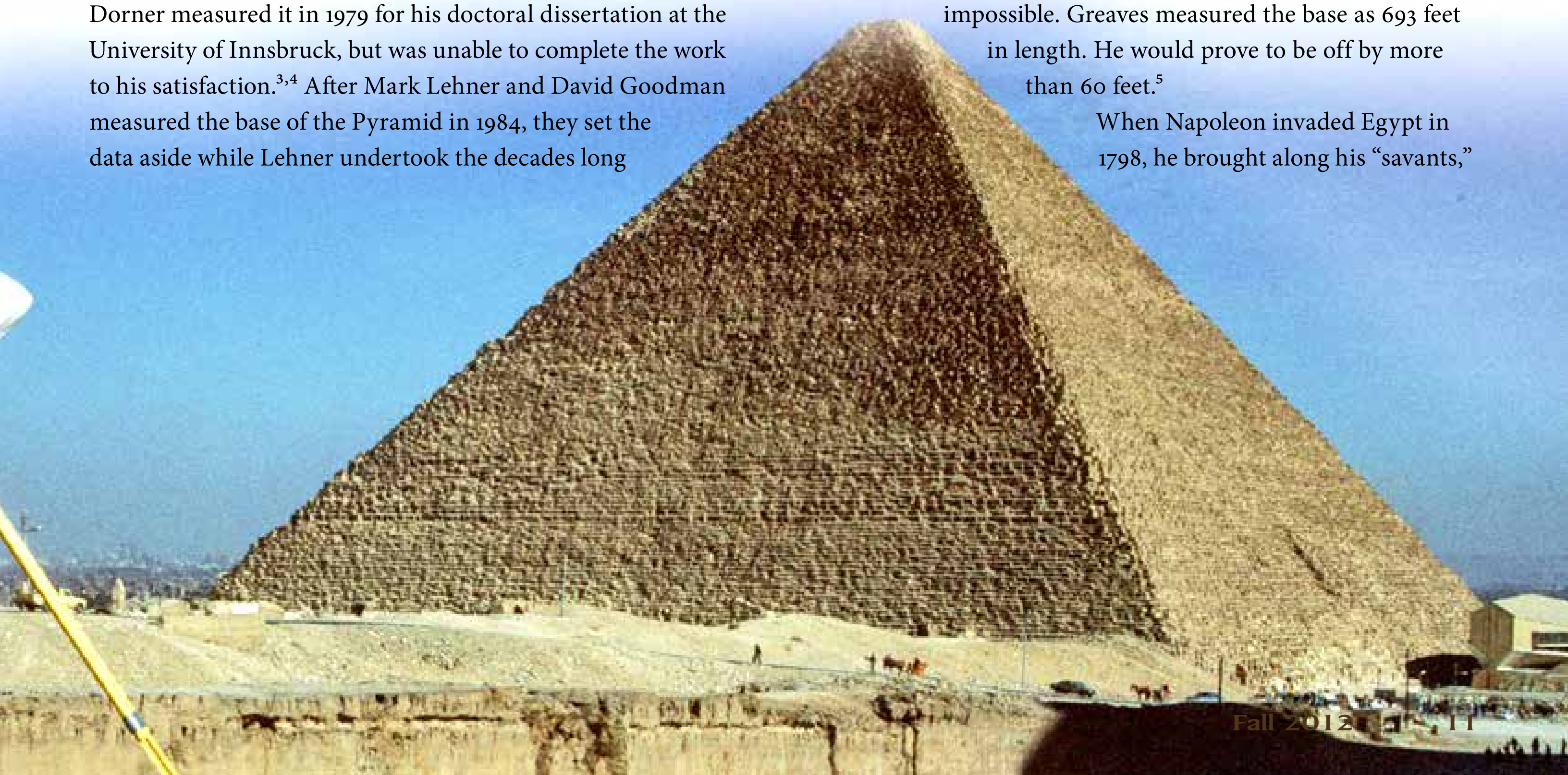
be a challenge. Of the original base, only 55 meters (180 feet) of what was once a casing baseline of 921 meters (3,022 feet) survives. Of the original platform baseline (as defined by its top, outer edge), only 212 meters (696 feet) of 924 meters (3031 feet) survives.

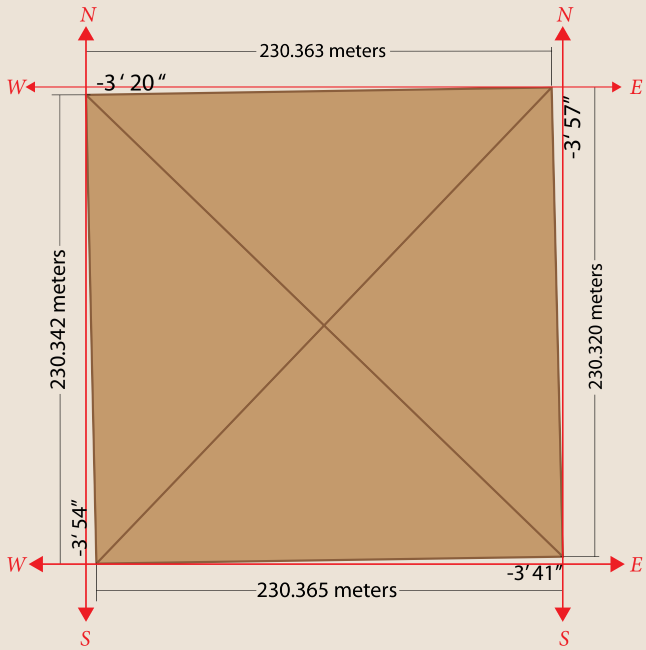
Flinders Petrie, the father of Egyptian archaeology, measured the base of the Great Pyramid from 1880 to 1882.[1] J. H. Cole, a surveyor with the Egyptian Ministry of Finance, made additional measurements, which he published in 1925.[2] Joseph Dorner measured it in 1979 for his doctoral dissertation at the University of Innsbruck, but was unable to complete the work to his satisfaction.[3,4] After Mark Lehner and David Goodman measured the base of the Pyramid in 1984, they set the data aside while Lehner undertook the decades long task of uncovering and mapping the Lost City of the Pyramids. I now return to it.

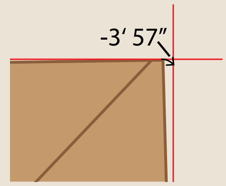
Pyramid Surveys: From Savants to the 1970s

John Greaves, Professor of Astronomy at the University of Oxford, made one of the first attempts in modern times to precisely measure the base of the Great Pyramid. However, upon his arrival in Egypt in 1638 he found the base covered in centuries-old debris. Accurate measurements were all but impossible. Greaves measured the base as 693 feet in length. He would prove to be off by more than 60 feet.[5]

When Napoleon invaded Egypt in 1798, he brought along his "savants,"







Detail of corner

PETRIE'S GREAT PYRAMID MEASUREMENTS

Side	Length (meters)	Angle
North	230.363	-3′ 20″
East	230.320	-3′ 57″
South	230.365	-3′ 41″
West	230.342	-3′ 54″
Average	230.348	-3′ 43″

This drawing and table present Petrie's measurements of the sides of the Great Pyramid and the angles at which they deviate from the cardinal directions. The drawing exaggerates the angles in order to display them. Petrie found that each side was rotated slightly counterclockwise from cardinal points, as indicated by the minus sign.

150 members of the "Commission of Arts and Sciences" to study and document sites throughout Egypt. Savant Edme-François Jomard assaulted the accumulated debris on the base of the Pyramid in Napoleonic style with a small army of Ottoman Turks. They cut through the overburden, uncovering two "sockets" off the northeast and northwest corners, one of which can be seen in the photos on the previous page. Jomard believed these sockets once held the very cornerstones of the Pyramid. To compute the Pyramid's orientation and size, he thought, one only needed to measure the relative positions of the socket's outermost corners.

Petrie's Measurements. Flinders Petrie, who arrived at Giza in 1880 to perform his measurements, disagreed with Jomard. By analyzing the Pyramid's angles, he determined that the true corners must have fallen somewhere inside the sockets. Petrie, at 27, had already gained recognition for his skills as a surveyor, even before winning lasting fame as an archaeologist.

By then, all four corner sockets had been found and exposed. Conveniently, Royal Astronomer and surveyor David Gill had preceded Petrie and in 1874 set bronze survey markers just inside the socket corners (shown in photo on previous page). Petrie, and almost every surveyor since, would use Gill's markers as control points.

Petrie found the north side of the Pyramid partially cleared of debris, revealing its ancient casing of smooth, white Tura limestone, seen in the photo on the facing page. The casing's outer surface, Petrie estimated, sloped at a mean angle of 51° and 52 minutes plus or minus 2 minutes. The casing once covered the entire Pyramid, requiring 21 acres of casing stones in all. Most of the casing had been carted away for building material centuries before. Originally it was supported by platform slabs set into bedrock, which Petrie found to be remarkably level.

At that time, however, most of the east, west, and south sides of the Pyramid still remained covered in debris. Petrie cut through the debris to find a section of well-preserved casing near the center of each side. He chose one point on each side and then measured their relative positions precisely.

Petrie then set out to calculate the size and orientation of the Pyramid by making a key assumption. He assumed that the corners of the Pyramid's casings fell on the "pyramid diagonals"—lines that connected the four socket corners to their opposing corner.

With that assumption and his measurements in hand, Petrie claimed he could calculate the length of the casing's baseline on each side and the orientation of each baseline relative to cardinal points using a complex geometrical argument.[6] He found that each side was rotated slightly counterclockwise from cardinal points, as indicated by the minus sign in angle measurements shown above in the table and the stylized pyramid with Petrie's measurements. The maximum difference in length between any two sides, Petrie said, was just 4.5 centimeters (about 1.75 inches), and the corners of the casing formed nearly perfect right angles. The maximum deviation from a 90° angle at any corner was at the northeast corner, where it was just 37 seconds of arc (.01°)—about the angle subtended by a dime viewed from across a football field.

However, as noted, Petrie measured just one point on each side. Establishing a line, of course, requires at least two.

Cole's Lines. In the Pyramid's case, no lines would actually be measured until the 1920s, when J. H. Cole of the Computation Office of the Egyptian Ministry of Finance did so at the request of the German archaeologist Ludwig Borchardt. Cole laboriously cut through debris to expose several more points of the casing on each side. He chose the "best" two on each

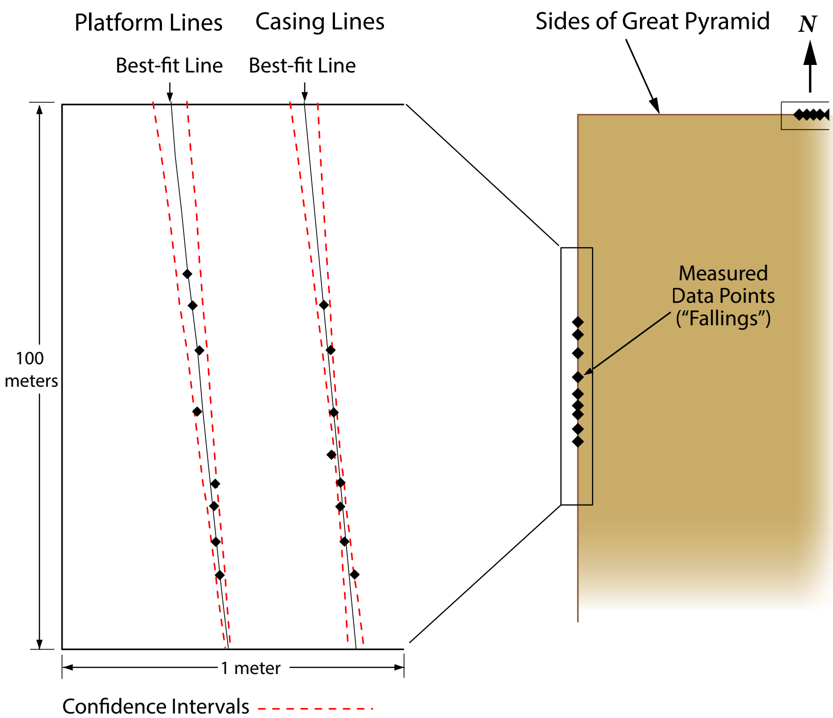
COLE'S GREAT PYRAMID MEASUREMENTS

Side	Length (meters)	Angle
North	230.353	-2′ 28″
East	230.391	-5′ 30″
South	230.454	-1′ 57″
West	230.357	-2′ 30″
Average	230.364	-3′ 06″

DORNER'S GREAT PYRAMID MEASUREMENTS

Side	Length (meters)	Angle
North	230.328	-2′ 28″
East	230.369	-3′ 26″
South	230.372	-2′ 31″
West	230.372	-2′ 47″
Average	230.360	-2′ 48″

Casing and Platform Stones. Near the corner of the north side the angled casing stones sit upon platform stones. The lower, outer edge of the casing and the top, outer edge of the platform provide the best places to measure the Pyramid's lines. Petrie estimated the angle of the casing slope to be 51° 52 minutes ± 2 minutes. Photo by Mark Lehner.



Lehner's Fallings. Mark Lehner mapped points near the middle of the east, west, and north side of the Great Pyramid where he found well preserved edges. We derive best-fit lines and confidence intervals for these. (The horizontal scale is exaggerated here to emphasize the angle.) The original corners can be located by extrapolation.

and measured their angle. His measurements for the Great Pyramid are shown in the table above.

The Pyramid was looking a little less perfect than what Petrie had determined. The maximum difference between any two sides, according to Cole, was 10 centimeters, about twice what Petrie had found. Its sides were also less square, with a deviation of about 3½ minutes of arc (.058°) at the northeast corner, about six times what Petrie found. Cole did find Petrie correct in one respect; the corners of the casing seemed to fall on the Pyramid diagonals.

The Egyptian Government eventually cleared the entire base of the Pyramid, but there would be no additional surveys until decades later.

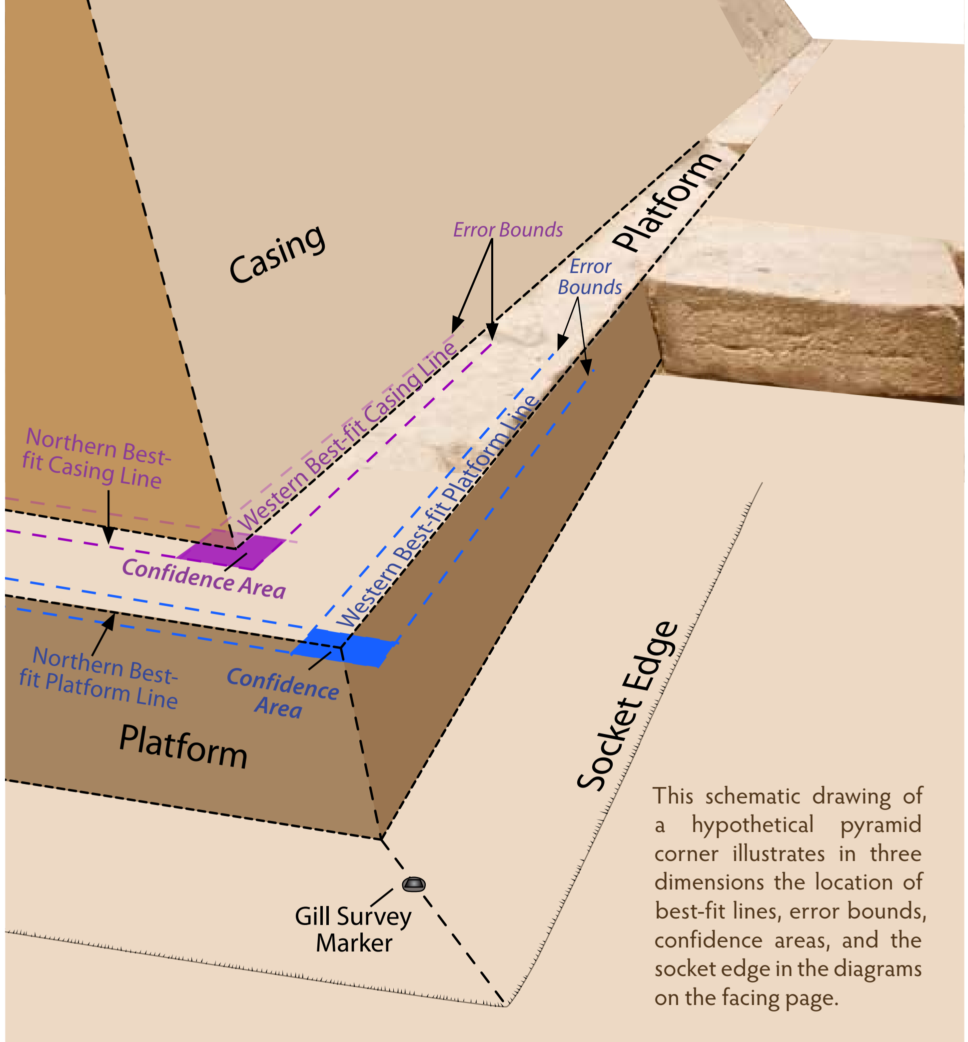
Dorner's Values. In 1979, when Josef Dorner surveyed the Great Pyramid for his doctoral dissertation, he was able to provide preliminary measurements for the Great Pyramid, as shown in the table above on the left.

The maximum difference between any two sides, according to Dorner, was 4.4 centimeters (almost 1.75 inches). The most askew of the right angles was 58 seconds (.016°) from square on the northeast. While not as perfect a pyramid as Petrie had proposed, Dorner's findings were more in line with Petrie's than Cole's.

Lehner's Fallings

In 1984, Mark Lehner and David Goodman made a comprehensive survey of the base of the Pyramid. Goodman, a surveyor then with the California Department of Transportation, established the survey grids now used to map both the Giza Plateau and the Valley of the Kings. For this study, he first laid a survey line along each side of the Pyramid between the bronze survey markers left by Gill, to serve as a control. Lehner then walked along the survey lines, choosing points to measure. When he chose a point, Goodman recorded its distance from one of Gill's markers electronically. Goodman then sighted along the survey line using his theodolite's telescope.

Lehner laid a tape measure from the point he wished to measure to the survey line, while Goodman, who could see the tape measure in his telescope, recorded the distance between the two. Surveyors refer to these offset measures as "fallings." At each station, Lehner carefully noted the condition of the edges of the casing and platform stones. Mapping those points where he found the top, outer edge of the platform stones or the lower edge of the casing stones well preserved, I can attempt to reconstruct the original lines of the Pyramid. While previous surveyors had concentrated only on the casing, Lehner measured the platform as well.



This schematic drawing of a hypothetical pyramid corner illustrates in three dimensions the location of best-fit lines, error bounds, confidence areas, and the socket edge in the diagrams on the facing page.

Analyzing the Lehner-Goodman Data

In order to analyze this data, I first need to place it on a master grid. The grid I will use is the Giza Plateau Mapping Project (GPMP) control network that was established by Lehner and Goodman in 1984 and 1985. It assigns every point on the plateau coordinates, like addresses for houses on a city map. The origin of the map lies at the calculated center of the Great Pyramid, and everything is measured from that point, in units of meters. For example, Gill's bronze survey marker off the northeast corner of the Pyramid is at 115.802 meters north of the center of the Pyramid, and 115.607 meters to its east. By convention, surveyors do not work with negative numbers, so instead of making the center of the Great Pyramid point (0, 0), Goodman and Lehner arbitrarily assigned it a location of (N100,000, E500,000). That places the northeast Gill marker at "Northing" 100,115.802 and "Easting" 500,115.607. As designed, the GPMP system can be used to map features up to 100 kilometers south of the Pyramid, and 500 kilometers to its west, with unlimited range to its north and east.

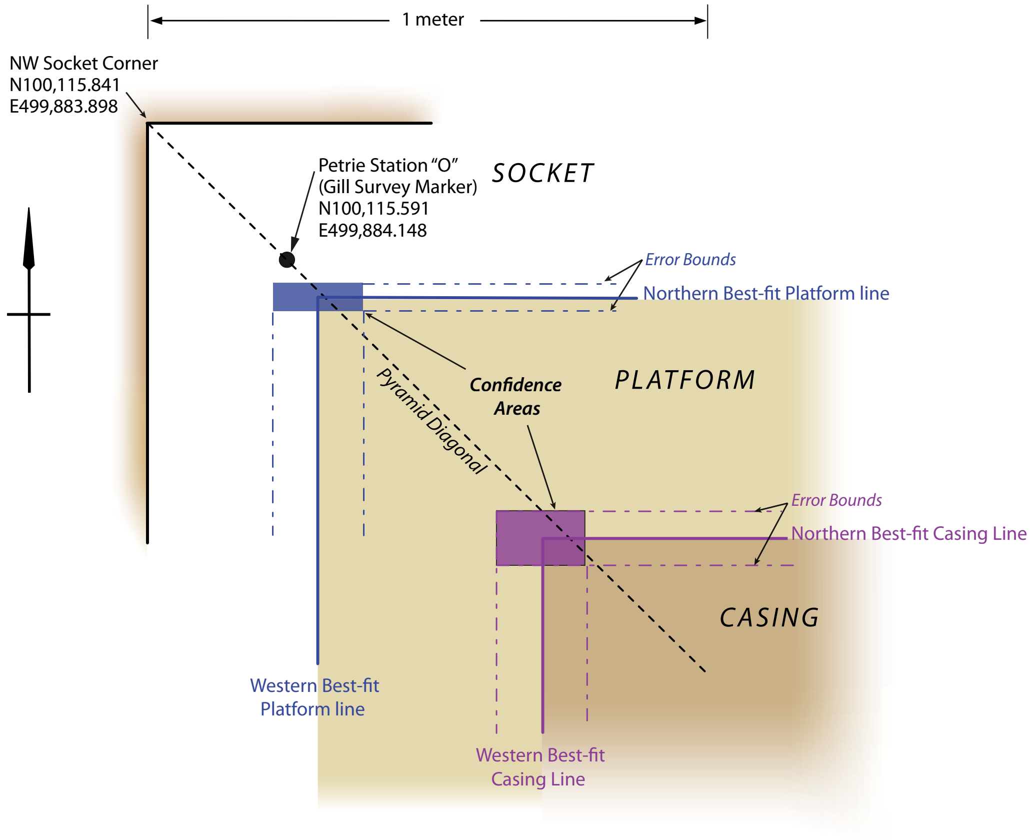
Once the Lehner-Goodman data is converted to GPMP coordinates, I can use a standard statistical method known as linear regression analysis to "best-fit" lines to it. In the figure on page 13 (center left), I show best-fit lines for the casing and the platform on the west side of the Pyramid. My linear regression analysis not only generates best-fit lines, but margins of error as well, known as confidence intervals.

I have generated best-fit lines and confidence intervals for the north and east sides as well.* To derive corners, I need only to extrapolate these lines to see where they cross. The figure on the facing page shows the situation at the northwest corner. Here, two sets of best-fit lines for the casing, and two for the platform, meet. Each line is accompanied by confidence intervals. Based on my measurements and assumptions, there is a 95% probability that the original casing and platform edges fell within the regions bounded by the dotted lines. For the casing, the error range, or "confidence area," is approximately 16 by 9 centimeters (6.3 by 3.5 inches). For the platform, it is about 16 by 5 centimeters (6.3 by 2.0 inches).

Is there a way to narrow this confidence area further? I can assume, as did Petrie and Cole, that all four corners of the platform and casing fell on the Pyramid's diagonals (shown in the figure on the right). The locations of the Pyramid diagonals are well documented.† Since I only need the intersection of two lines to define a corner, and I am assuming that the casing and platform corners fell on the diagonal, I only need consider the intersection of either the northern casing and platform lines

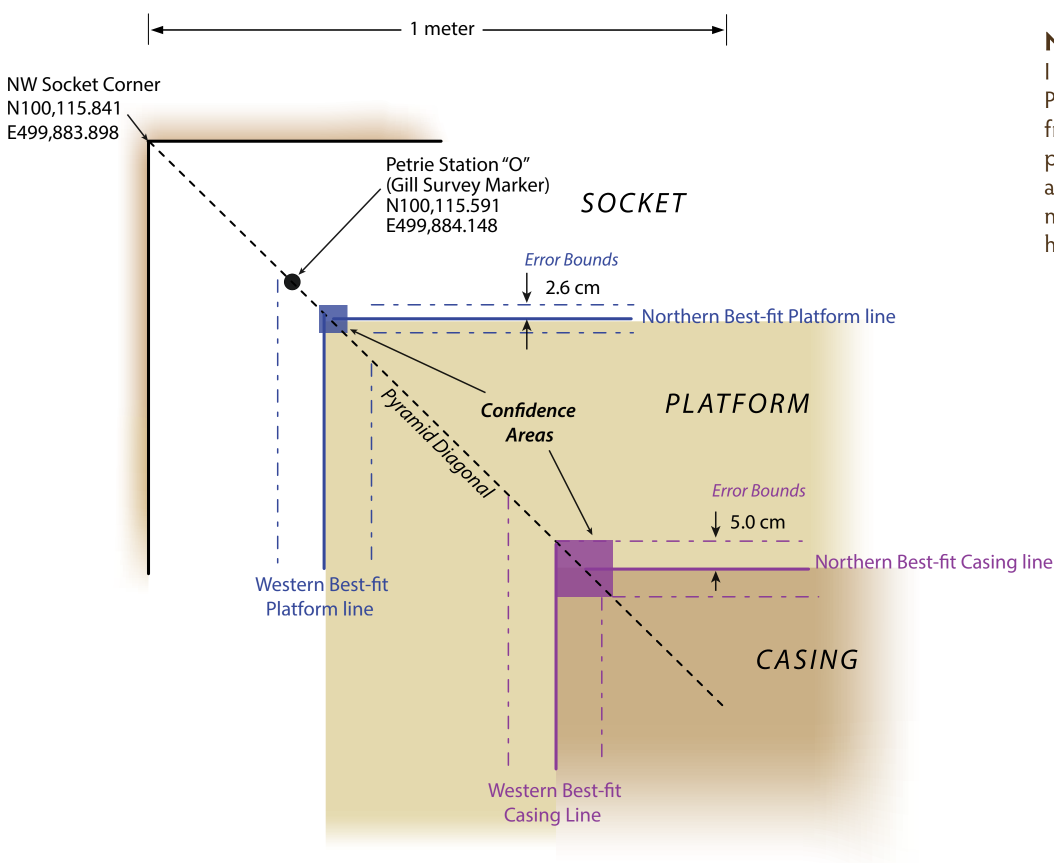
* The south side is too badly damaged to provide data useful for statistical analysis. However, since I am assuming that the original casing and platform corners fell on the diagonals, I can proceed without that data.

† Petrie and Maragioglio and Rinaldi placed the northern socket corners in exactly the same locations. Compare Plate x in W. M. F. Petrie, *The Pyramids and Temples of Gizeh*, London: Field and Tuer, 1883, with Plate 2 in V. Maragioglio and C. Rinaldi, *L'Architettura Delle Piramidi Menfite* Parte IV, Rapallo, Tipografia Canessa, 1965.



The Northwest Corners

I can locate the corners from the intersection of the best-fit lines derived from the Lehner-Goodman data. Each line is surrounded by confidence intervals. (While the confidence intervals are, in fact, curved [hyperbolic] they appear straight over short distances.) There is a 95% chance that the original corners fell within the "confidence areas."



Narrowing the Range

I can use the intersection of the Pyramid diagonal, which extends from the socket corner to the approximate center of the Pyramid, and the northern best fit lines, to narrow the confidence areas. This helps to better locate the corners.

LEHNER-GOODMAN CASING CORNERS

Corner	Northing	Easting	Confidence Areas (meters)
Northeast	100,115.288	500,115.034	±.054
Southeast	99,885.006	500,115.262	±.093
Southwest	99,884.759	499,884.954	±.060
Northwest	100,115.095	499,884.645	±.050

LEHNER-GOODMAN PLATFORM CORNERS

Corner	Northing	Easting	Confidence Areas (meters)
Northeast	100,115.668	500,115.414	±.013
Southeast	99,884.484	500,115.785	±.031
Southwest	99,884.396	499,884.592	±.023
Northwest	100,115.522	499,884.217	±.026

with the diagonal, or the intersection of the western casing and platform lines with the diagonal. The northern lines have narrower confidence intervals and thus are better defined. Therefore, I will locate the northwest platform and casing corners at the intersections of the northern lines with the diagonal. The regions bounding their intersections are their confidence areas.

Applying the same procedure at all four of the Pyramid's corners, I can derive their locations. In the tables above I provide my best estimates for the original locations of the corners and their confidence areas. The largest of the confidence areas is at the southeast, but even there I can locate the casing corner to within ± 9.3 centimeters (3.7 inches).

I can also use this data to calculate the length of the Pyramid's sides and its angles. The Lehner-Goodman estimates for the casing lengths compared with that of Petrie, Cole, and Dorner are in the table on the right. Petrie's and Dorner's measurements fit comfortably inside the Lehner-Goodman ranges. Lehner-Goodman and Petrie differ in the mean of all four sides by only 1.8 centimeters (0.75 inches). One of Cole's measurements, however, falls outside the Lehner-Goodman ranges (in italics).

As for the angles, the Lehner-Goodman estimates are compared with that of Petrie, Cole, and Dorner in the

THE GREAT PYRAMID'S CASING LENGTHS IN METERS: LEHNER-GOODMAN, PETRIE, COLE, AND DORNER

Side	Lehner/Goodman			Petrie	Cole	Dorner
	Min	Mean	Max			
North	230.286	230.389	230.493	230.363	***230.253***	230.328
East	230.135	230.282	230.429	230.320	230.391	230.369
South	230.155	230.309	230.462	230.365	230.454	230.372
West	230.227	230.337	230.447	230.342	230.357	230.372
Average		230.329		230.348	230.364	230.360

THE GREAT PYRAMID'S CASING ANGLES: LEHNER-GOODMAN, PETRIE, COLE, AND DORNER

Side	Lehner/Goodman			Petrie	Cole	Dorner
	Min	Mean	Max			
North	-1′ 19″	-2′ 52″	-4′ 25″	-3′ 20″	-2′ 28″	-2′ 28″
East	-1′ 12″	-3′ 24″	-5′ 36″	-3′ 57″	-5′ 30″	-3′ 26″
South	-1′ 24″	-3′ 41″	-5′ 58″	-3′ 41″	-1′ 57″	-2′ 31″
West	-2′ 58″	-4′ 37″	-6′ 14″	-3′ 54″	***-2′ 30″***	***-2′ 47″***
Average		-3′ 38″		-3′ 43″	-3′ 06″	-2′ 48″

LEHNER-GOODMAN LENGTH AND ANGLES OF THE GREAT PYRAMID'S PLATFORM

Side	Length (meters)			Angle		
	Min	Mean	Max	Min	Mean	Max
North	231.157	231.196	231.236	-1′ 35″	-2′ 10″	-2′ 45″
East	231.140	231.184	231.229	-4′ 51″	-5′ 31″	-6′ 11″
South	231.138	231.193	231.248	-29″	-1′ 18″	-2′ 07″
West	231.076	231.126	232.176	-4′ 50″	-5′ 34″	-6′ 18″
Average		231.175			-3′ 38″	

table on the left. All the measurements fall within the Lehner-Goodman ranges except for the Dorner and Cole measurements on the west side.‡

Recalling that Lehner measured the platform as well, I include its lengths and angles in the table on the left. The platform extends outward from the casing by an average of 42.3 centimeters (16.7 inches) on each side. The casing does not run quite parallel to the platform. Although this difference is too small to illustrate in our figures, it is still significant and helps us to understand how the Pyramid was built. It might suggest, for example, that the Pyramid's builders were unsatisfied with the platform's original lines and chose to square things up a bit before finally dressing the casing down.

‡ Dorner initially set his azimuth by measuring the angle of the casing on the north side with a WILD meridian telescope. He found the north side running at an angle of -3' 0." However, he rejected his own measurement in favor of Cole's: – 2' 28." My analysis indicates he would have been better off not doing so. If he had accepted his own measurements, all his angles would change by -32" of arc. Not only would these revised angles fall within the Lehner-Goodman ranges, but they would be quite close to Petrie's values as well.

The South Side

In this analysis, I managed to compute the length and orientation of the base of the Great Pyramid without the benefit of data from its south side. I was able to do this because I assumed that the corners of both the casing and the platform fell on the socket diagonals. This was necessary because so little of the south survives. There, the top, outer edge of the platform is nowhere to be found. As for the casing, at one point 122.2 meters east of the southwest Gill marker, Lehner found that the casing once met the platform at N99,884.838 and E500,006.828. My model predicts that at that location the casing should have fallen at N99,884.889 and E500,006.889 plus or minus 0.075 meters. The casing does indeed fall within the range my model predicts. It is the only usable data point on the casing I have identified for the south side.

Conclusions

I gather my results in the figure on the next page. I have derived new estimates for the locations of the casing and

Lost in the fog of time: the exact size and orientation of the Great Pyramid (above left) and the Khafre Pyramid (above) are not certain because the corners are missing. Right: One of the Lehner-Goodman 1984 survey sheets recording their fallings on the south side of the Great Pyramid. The detail shows Lehner's sketches of the casing and platform at three points. The center sketch includes the comments: "Casing foot extrapolated from extant face at top of block." "Plat. surface here slightly worn away. Note this upper piece shows a crack along the top – slightly dislodged."

platform corners and provided error bounds (confidence areas). I can fix the locations of the platform corners to within 4 centimeters, and the casing corners within 10 centimeters.

The mean of the Lehner-Goodman estimates for the casing corners are remarkably close to Petrie's. The largest deviation between the two is on the northwest and is less than 4 centimeters (1.6 inches).

ACKNOWLEDGEMENTS

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References

1. W. M. F. Petrie, *The Pyramids and Temples of Gizeh*, London: Field and Tuer, 1883.

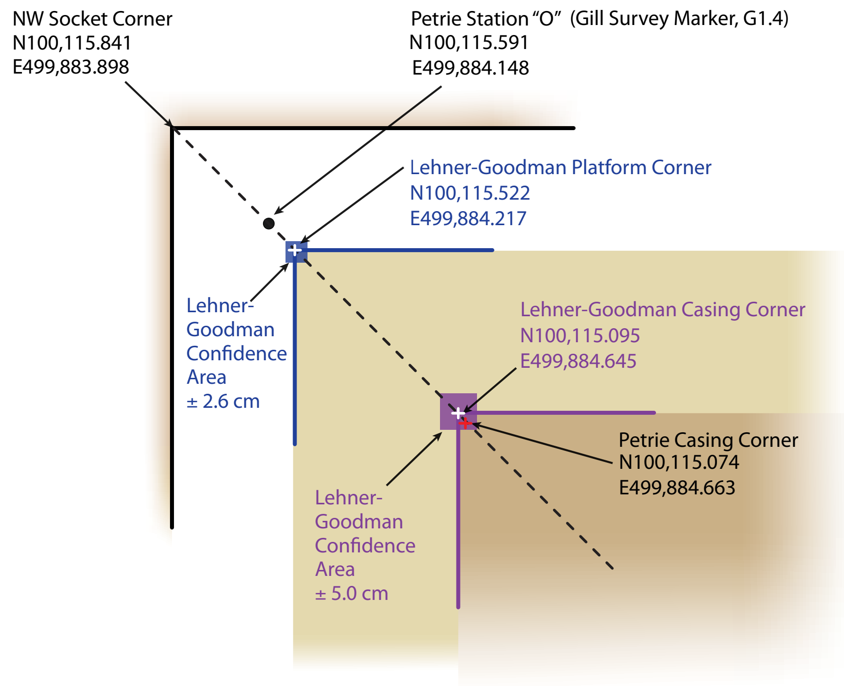
2. J. H. Cole, *Determination of the Exact Size and Orientation of the Great Pyramid of Giza*, Cairo: Government Press, 1925.

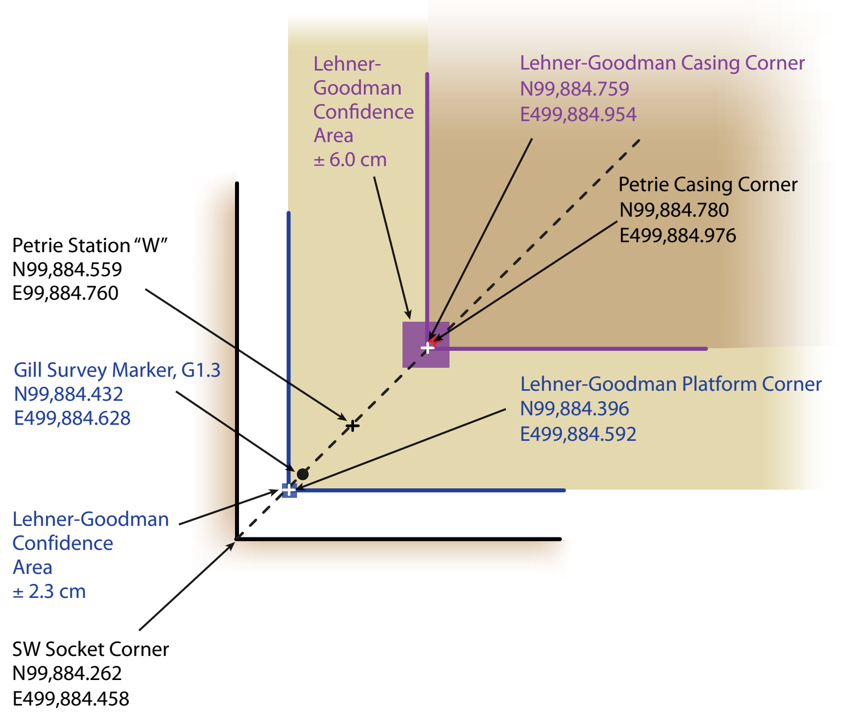
3. J. Dorner, Die Absteckung und astronomische Orientierung aegyptischer Pyramiden, PhD Dissertation, Universitaet Innsbruck, 1981.

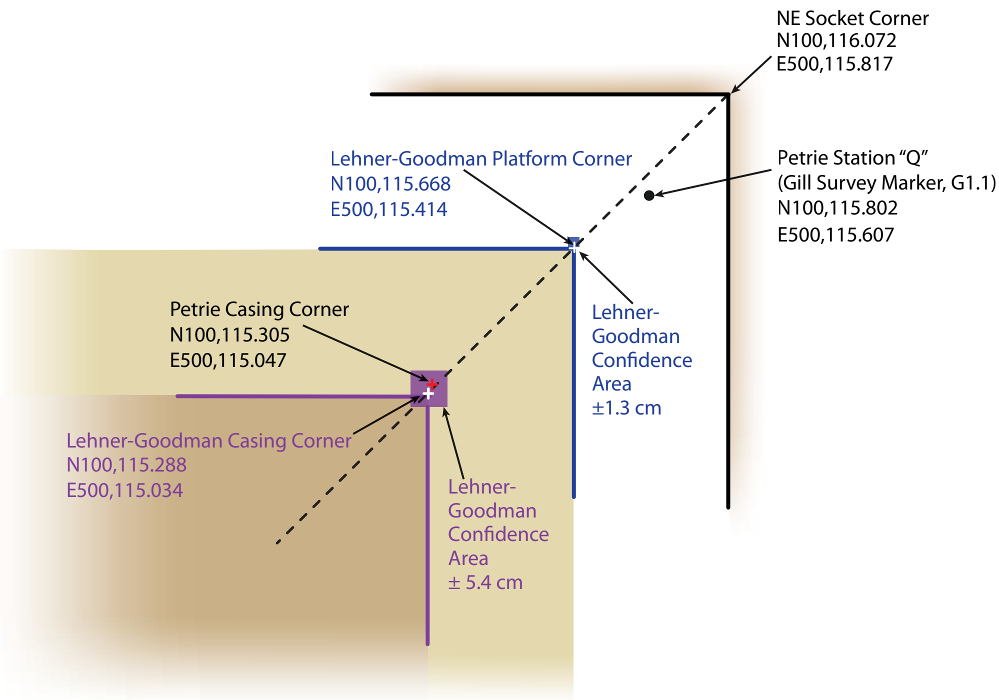
4. J. Dorner, "Das Basisviereck der Cheopspyramide," in *Structure and Significance, Thoughts on Ancient Egyptian Architecture*, P. Janosi, ed., Vienna: Verlag der Oestrreichischen Akademie der Wissenschaften, 2005.

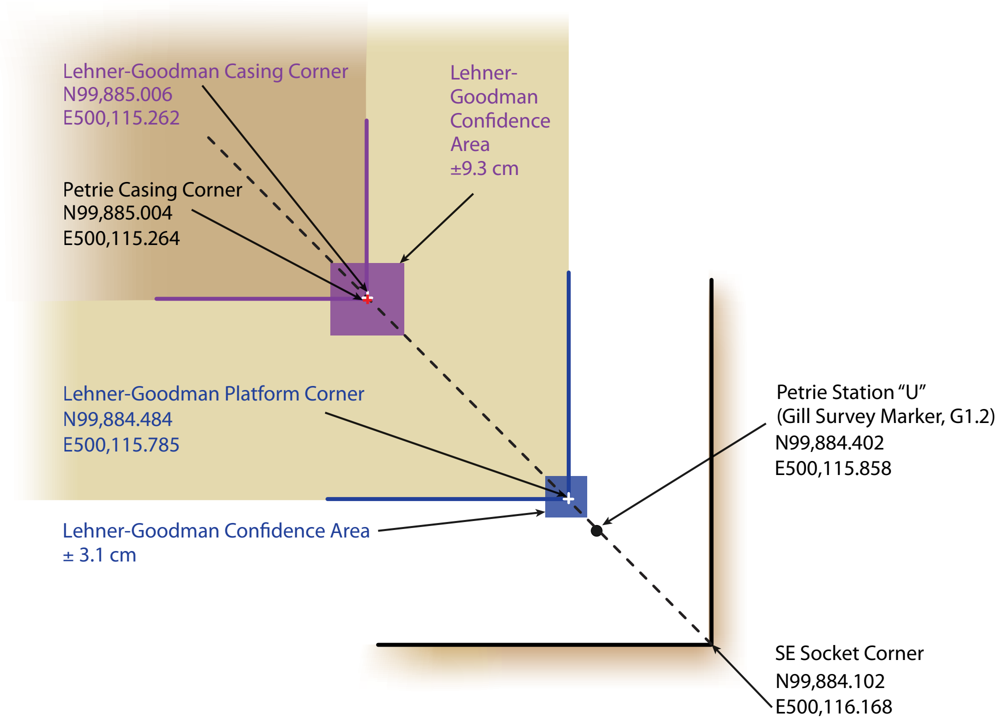
5. J. Greaves, *Pyramidographia, or, a Description of the Pyramids in Ægypt,* London: King's Arms, 1736.

6. W. M. F. Petrie, *The Pyramids and Temples of Gizeh*, London: Field and Tuer, 1883, page 39.









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